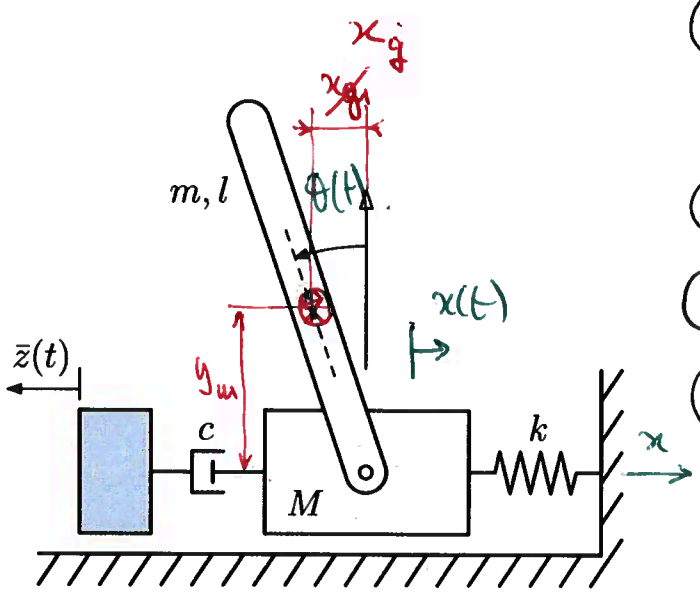
$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) - \frac{\partial T}{\partial x} + \frac{\partial V}{\partial x} = 0$$

$$\frac{1}{T} \cdot \frac{F \times L}{L/T} = F$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) - \frac{\partial T}{\partial \theta} - \frac{\partial V}{\partial \theta} = 0$$

$$\frac{1}{T} \frac{M \times A}{A/T} = M$$



1. Generalised coordinates
   $x(t)$ and $\theta(t)$
2. kinetic energy T
3. potential energy V
4. Lagrange equations

2. KE : $T = T_M + T_{\theta_m} + T_{x_m} + T_{y_m}$

mass m: T_m

$$\begin{cases} T_M = \frac{1}{2} M \dot{x}^2 \\ T_m = \frac{1}{2} I_G \dot{\theta}^2 + \frac{1}{2} m \left(\dot{x}_m^2 + \dot{y}_m^2 \right) \end{cases}$$

$$\begin{cases} x_m = \frac{1}{2} l \sin\theta + x \\ y_m = \frac{1}{2} l \cos\theta \end{cases} \Rightarrow \begin{cases} \dot{x}_m = \dot{x} + \frac{1}{2} l \dot{\theta} \cos\theta \\ \dot{y}_m = -\frac{1}{2} l \dot{\theta} \sin\theta \end{cases} \quad \begin{cases} \dot{x}_m^2 = \dots \\ \dot{y}_m^2 = \dots \end{cases}$$

$$T = \frac{1}{2}(M+m)\dot{x}^2 + \frac{1}{2} m l \dot{x} \dot{\theta} \cos\theta + \frac{1}{6} m l^2 \dot{\theta}^2$$

3. $V = V_e + V_g = \frac{1}{2} k x^2 + \frac{1}{2} m g l \cos\theta$

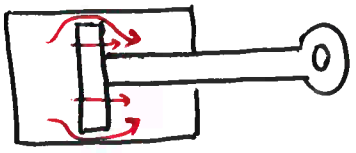
V_e: spring; V_g: g (NEVER neglect)

Lecture 6
8-Oct

④

EoM x: $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) - \frac{\partial T}{\partial x} + \frac{\partial V}{\partial x} = Q_x = ?$ (*) (L12)

EoM θ: $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) - \frac{\partial T}{\partial \theta} + \frac{\partial V}{\partial \theta} = Q_\theta = ? = 0$



$F_d (= -c\dot{x})$ (*) $= -c\left|\left(\dot{z} - \dot{x}\right)\right|$

$F_s = kx$

EoM (x): $\frac{\partial T}{\partial \dot{x}} = (M+m)\dot{x} + \frac{1}{2}ml\dot{\theta}\cos\theta$

$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) = (M+m)\ddot{x} + \frac{1}{2}ml\ddot{\theta}\cos\theta - \frac{1}{2}ml\dot{\theta}\dot{\theta}\sin\theta$

$\frac{\partial T}{\partial x} = 0$; $\frac{\partial V}{\partial x} = kx$; $Q_x = -c\left|\dot{z} - \dot{x}\right|$

EoM (x) $(M+m)\ddot{x} + \frac{1}{2}ml\ddot{\theta}\cos\theta - \frac{1}{2}\dot{\theta}^2 ml\sin\theta + kx = -c\left|\dot{z} - \dot{x}\right|$

($\cos\theta \approx 1$; $\dot{\theta}^2 = ?$; $\sin\theta \approx \theta$)

EoM (θ) $\frac{\partial T}{\partial \dot{\theta}} = \frac{1}{2}ml\dot{x}\cos\theta + \frac{1}{3}ml^2\dot{\theta}$

$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) = \frac{1}{2}ml\ddot{x}\cos\theta - \frac{1}{2}ml\dot{x}\dot{\theta}\sin\theta + \frac{1}{3}ml^2\ddot{\theta}$

$\frac{\partial T}{\partial \theta} = -\frac{1}{2}ml\dot{x}\dot{\theta}\sin\theta$; $\frac{\partial V}{\partial \theta} = -\frac{1}{2}mgl\sin\theta$

EoM (θ) $\frac{1}{3}ml^2\ddot{\theta} + \frac{1}{2}ml\ddot{x}\cos\theta - \frac{1}{2}mgl\sin\theta = 0$

($\cos\theta \approx 1$; $\sin\theta \approx \theta$)

LM3

Lecture 7
13-Oct-2026

≈ 0

EoM (x) $\quad (M+m)\ddot{x} + \frac{1}{2}ml\ddot{\theta} - \frac{1}{2}\dot{\theta}^2 ml\theta + kx = -c\left|\dot{\frac{l}{2}}\theta - \dot{x}\right|$

EoM (θ) $\quad \frac{1}{3}ml^2\ddot{\theta} + \frac{1}{2}ml\ddot{x} - \frac{1}{2}mgl\theta = 0$

EoM (x) $\quad (M+m)\ddot{x} + \frac{1}{2}ml\ddot{\theta} + kx = -c\left|\dot{\frac{l}{2}}\theta - \dot{x}\right|$